Parent Support Sheet – Maths – Upper Key Stage 2

At Yewtree, we follow Herts for Learning (HfL) Essentials Written Calculation Progression, which links the key concrete experiences with pictorial and abstract representations (written symbolic and spoken). This supports pupils to move with confidence and deep conceptual understanding through each strand of their calculation. Below is a summary of what concrete, pictorial and abstract representations are:

Concrete manipulatives - are objects that can be touched and moved by pupils to introduce, explore or reinforce a mathematical concept. They provide a vehicle to help pupils make sense of complex, symbolic and abstract ideas through exploration and manipulation. They also support the development of internal models and help build stronger memory pathways.

Pictorial (including jottings) - the act of translating the concrete experience into a pictorial representation helps focus attention on what has happened and why. This supports deeper understanding and a stronger imprint on memory. Pictorial representations are more flexible than concrete resources and, once understanding is secured, allow exploration of complex problems that may be challenging to reproduce with manipulatives (resources).

Abstract –Written forms of notation. These have developed through the history of mathematics. Clear individual steps in procedure are hidden or they have been shortcut. The informal and expanded methods highlight all the intermediate steps, repeating thought processes more closely and support understanding prior to compaction of the forms of notation.

Abstract - Spoken - learning to use the correct mathematical vocabulary is vital for the development of mathematical proficiency. The ability to articulate accurately allows pupils to communicate and build meaning. Ideas become more permanent. This can be constructed using speaking frames.

As set out by the National Curriculum – Maths Programme of Study, the principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems. Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

Pupils should read, spell and pronounce mathematical vocabulary correctly.

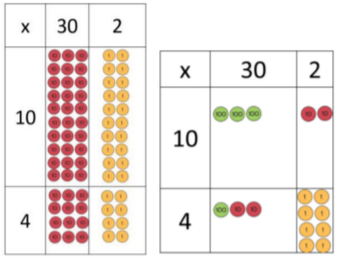
At the end of Year 6, pupils will sit national summative assessments, known as SATs.

The examples below show what is expected in upper key stage 2 when pupils are adding, subtracting, multiplying and dividing using the concrete, pictorial and abstract methods. When talking through these operations with your child, please refer to these methods. If you need any further help or clarification please speak to your child's teacher. Please also refer to years 5 and 6's long-term plans, programme of study and the key concept and vocabulary maps, which can all be found under the 'Curriculum' section on the school website. These documents will show you the topics your child is learning and will ensure your child is familiar with the appropriate terminology for the relevant topics.

Please refer to the Lower Key Stage 2 Parent support sheet for addition and subtraction methods, the short multiplication method and other key mathematical topics taught lower down the school.

This example shows the expanded vertical multiplication method – 2-digit by 2-digit -

Concrete



Pictorial - Jottings

x	30	2	
10	300	20	= 320
4	120	8	= 128

Abstract - Written symbolic

```
   3 2
 x 1 4
     8
 1 2 0
   2 0
 3 0 0
 4 4 8
```

32 x 14 = 448

Abstract - Speaking frame

First, I need to consider the ones in the multiplier.
… groups of … ones is ones.
… groups of … tens is tens. *(Do I need to regroup?)*
Then, tens in the multiplier.
… groups of … ones is ones. *(Do I need to regroup?)*
… groups of … tens is tens. *(Do I need to regroup?)*
The total of all the partial products is … .
The product of … and … is … .

Notes:

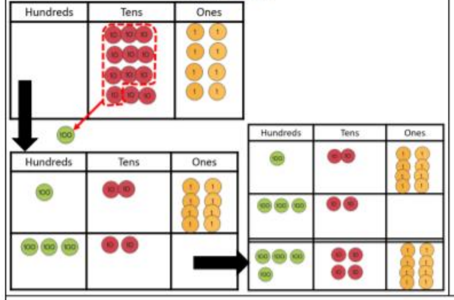
This is a transitional method towards long multiplication. Using the grid supports pupils in their thinking about multiplying by powers of ten and place value. Secure understanding of both of these concepts allow pupils to move to long multiplication more successfully.

Speaking frame hint: linking to what we know and correct place value. For example, 10 groups of 3 tens is 30 tens. This can be regrouped to 3 hundreds.

Multiplication

This example shows long multiplication 2-digit by 2-digit with simple re-grouping -

Concrete



Pictorial - Jottings

x	30	2	
10	300	20	= 320
4	120	8	= 128

Abstract - Written symbolic

```
   3 2
 x 1 4
 1 2 8
 3 2 0
 4 4 8
```

32 x 14 = 448

Abstract - Speaking frame

First, I need to consider the ones in the multiplier.
… groups of … ones is ones. (Do I need to regroup?)
… groups of … tens is tens. (Do I need to regroup?)
Then, considering tens in the multiplier.
… groups of … ones is ones. (Do I need to regroup?)
… groups of … tens is tens. (Do I need to regroup?)
The total of all the partial products is … .
The product of … and … is … .

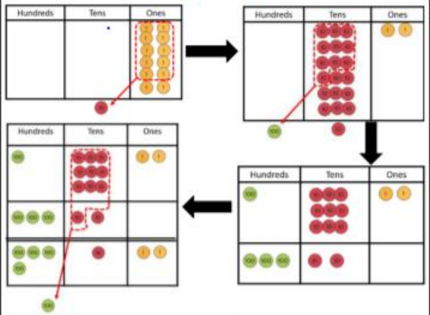
Notes:

Speaking frame hint: linking to what we know and correct place value.
For example, 10 groups of 3 tens is 30 tens (linking to known fact 10 x 3). This can be regrouped to 3 hundreds.

Multiplication

This example shows the long multiplication 2-digit by 2-digit, focusing on regroup in first partial product -

Concrete



Pictorial

x	30	2	
10	300	20	= 320
6	180	12	= 192

Abstract - Written symbolic

```
   1
   3 2
 x 1 6
 1 9 2
 3 2 0
 5 1 2
 1
```

32 x 16 = 512

Abstract - Speaking frame

First, I need to consider the ones in the multiplier.
… groups of … ones is … ones. *(Do I need to regroup?)*
… groups of … tens is … tens. *(Any regroups to add? Do I need to regroup?)*
Then, considering tens in the multiplier.
… groups of … ones is … ones. *(Do I need to regroup?)*
… groups of … tens is … tens. *(Do I need to regroup?)*
The total of all the partial products is … .
The product of … and … is … .

Notes:

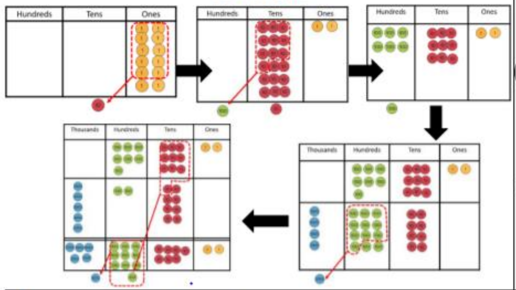
Speaking frame hint: linking to what we know and correct place value.

For example, 6 groups of 3 tens is 18 tens (linking to known fact 6 x 3 = 18). This can be regrouped to 1 hundred and 8 tens.

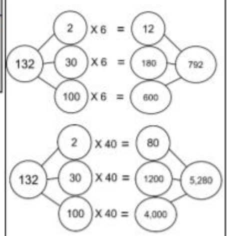
Multiplication

This example shows long multiplication 2-digit by 2-digit regrouping in first and second stage -

Concrete



Pictorial - Jottings



Abstract - Written symbolic

```
  1
  1 1
  1 3 2
    x 4 6
    7 9 2
  5 2 8 0
  6 0 7 2
  1 1
```

132 x 46 = 6,072

Abstract - Speaking frame

First, I need to consider the ones in the multiplier.
… groups of … ones is … ones. *(Do I need to regroup?)*
… groups of … tens is … tens. *(Any regroups to add? Do I need to regroup?)*
Then, considering tens in the multiplier.
… groups of … ones is … ones. *(Do I need to regroup?)*
… groups of … tens is … tens. *(Any regroups to add? Do I need to regroup?)*
The total of all the partial products is … .
The product of … and … is … .

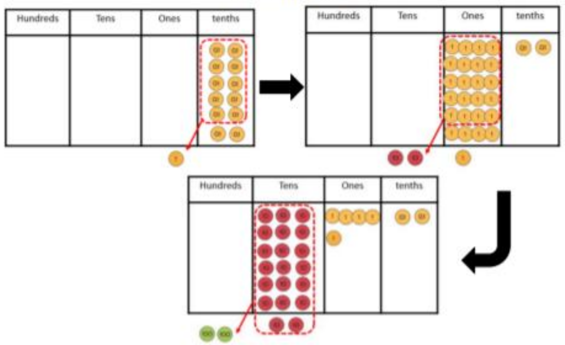
Notes:

Speaking frame hint: linking to what we know and correct place value.
For example, 6 groups of 3 tens is 18 tens (linking to known fact 6 x 3 = 18). This can be regrouped to 1 hundred and 8 tens.

Multiplication

This example shows formal written multiplication involving numbers with up to 2 decimal places multiplied by a 1-digit number -

Concrete



Pictorial - Jottings

Jottings: multiples of tricky multipliers

6
12
18
24
30
36
42
48
54
60
66
72

Abstract - Written symbolic

```
 2 1
 34. 2
x     6
-------
205. 2
```

34.2 x 6 = 205.2

Abstract - Speaking frame

… groups of … tenths is … tenths. (Do I need to regroup?)
… groups of … ones is … ones. *(Any regroups to add? Do I need to regroup?)*
… groups of … tens is … tens. *(Any regroups to add? Do I need to regroup?)*
The product of … and … is … .

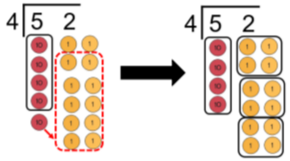
Notes:

Speaking frame hint: linking to what we know and correct place value.
For example, 6 groups of 3 tens is 18 tens (linking to known fact 6 x 3 = 18). This can be regrouped to 1 hundred and 8 tens.

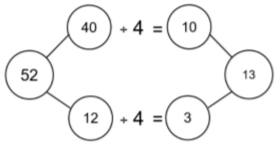
Multiplication

This example introduces formal short division regroup from tens to ones (grouping structure) -

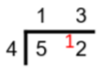
Concrete



Pictorial - Jottings



Abstract - Written symbolic



52 ÷ 4 = 13

Abstract - Speaking frame

I want to know how many groups of … are in … .
How many groups of … tens are in … tens without regrouping?
I can make … group(s) of … tens. There is/are … ten(s) remaining.
I need to regroup the … tens into … ones.
I now have … ones.
How many groups of … ones are in … ones, without regrouping?
I can make … group(s) of … ones. There is/are … one(s) remaining.

There are … groups of … in … with … remainders.

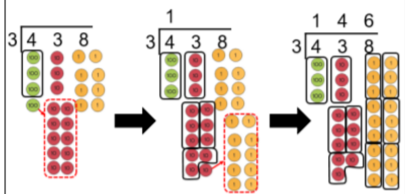
Notes:

Pupils are encouraged to progress to a grouping model of division. This is in preparation for 2-digit divisors and understanding fractions expressed as part of the quotient.
Pupils should explore with simple division calculations to ensure that they understand the shift in structure.
Speaking frame note: In this example, the speaking frame would be completed like this: *"How many groups of 3 tens are in 4 tens, without regrouping?"*
This is to ensure that accurate place value and magnitude is maintained.

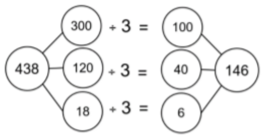
Division

This example shows short division for numbers up to 4-digits (grouping structure) -

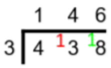
Concrete



Pictorial - Jottings



Abstract - Written symbolic



438 ÷ 3 = 146

Abstract - Speaking frame

I want to know how many groups of … are in … .
How many groups of … hundreds are in … hundreds, without regrouping?
I can make … group(s) of …hundreds. There is/are … hundred(s) remaining.
I need to regroup the … hundreds into …tens.

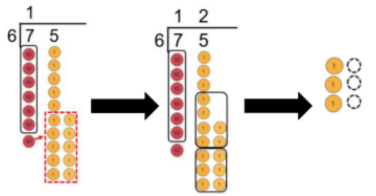
Notes:

Speaking frame note: This is an extension to the previous speaking frame.
In this example, the speaking frame would be completed like this:
"How many groups of 3 hundreds are in 4 hundreds, without regrouping?"
This is to ensure that accurate place value and magnitude is maintained.

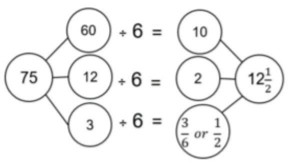
Division

This example shows short division (grouping structure) - expressing quotients with fractions -

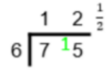
Concrete



Pictorial



Abstract - Written symbolic



$75 \div 6 = 12\frac{1}{2}$

Abstract - Speaking frame

I have a remainder of … .
This is … (remainder) out of … (divisor) which I need for another group.
This can be written as a fraction — .
This can be simplified to — .

Notes:

Speaking frame note: This is an extension to the previous speaking frame (5LS12 Step 2). In this example the speaking frame would be completed like this:

"I have a remainder of 3.

This is 3 out of 6 which I need for another group.

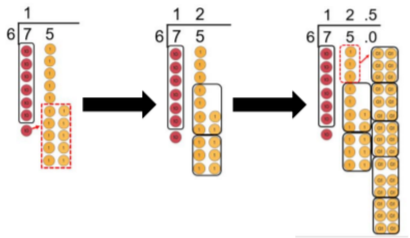
This can be written as a fraction $\frac{3}{6}$.

This can be simplified to $\frac{1}{2}$."

Division

This example shows short division (grouping structure) - expressing quotients with decimals -

Concrete

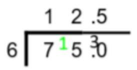


Pictorial - Jottings

Jottings: multiples of the divisor

6
12
18
24
30
36
42
48
54
60

Abstract - Written symbolic



$75 \div 6 = 12.5$

Abstract - Speaking frame

I have a remainder of … .
I need to regroup the … ones into … tenths.
How many groups of … tenths are in … tenths, without regrouping?
I can make … group(s) of … tenths.

There are … groups of … in … .

Notes:

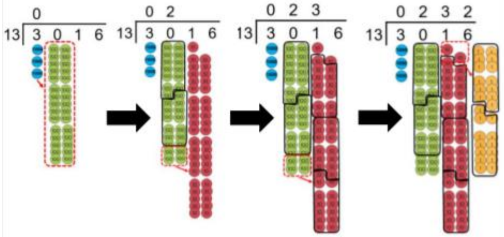
Speaking frame note: This is an extension to the previous speaking frame (5LS12 Step 2). In this example, the speaking frame would be completed like this:

"I have a remainder of 3.
I need to regroup the 3 ones into 30 tenths.
How many groups of 6 tenths are in 30 tenths, without regrouping?
I can make 5 groups of 6 tenths.
There are 12.5 groups of 6 in 75."

Division

This example shows long division for numbers up to 4 digits -

Concrete

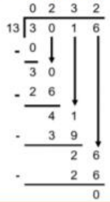


Pictorial - Jottings

Jottings: multiples of the divisor

13
26
39
52
65
78
91
104

Abstract - Written symbolic



3016 ÷ 13 = 232

Division

Abstract - Speaking frame

I want to know how many groups of … are in … .
How many groups of … thousand are in …thousand, without regrouping?
I can make … group(s) of …thousand. There is/are … thousand(s) remaining.
I need to regroup the … thousand(s) into …hundreds.

Notes:

The structure of long division was first introduced in 3LS30, then revisited and extended in both years 4 and 5. It was revised in Step 1 of this sequence.
Jottings are used to scaffold to derived related division facts.
Speaking frame note: This is an extension to the previous speaking frame (5LS12 Step 2). In this example, the speaking frame would be completed like this:
"How many groups of 13 thousands are in 3 thousand, without regrouping?" I can make zero groups of 13 thousand. There are 3 thousand remaining. I need to regroup the 3 thousands into 30 hundreds."

<u>Additional Year 6 examples –</u>

This example shows long multiplication; up to 4-digit by 2-digit -

Abstract speaking frame

First, I need to consider the ones in the multiplier.
7 groups of 6 ones is 42 ones.
I need to regroup into 4 tens and 2 ones.
7 groups of 3 tens is 21 tens.
I need to add the regrouped 4 tens. I now have 25 tens.
I need to regroup into 2 hundreds and 5 tens.
7 groups of 8 hundreds is 56 hundreds.
I need to add the regrouped 2 hundreds. I now have 58 hundreds. I can regroup this into 5 thousands and 8 hundreds.

Then, considering the tens in the multiplier.
20 groups of 6 ones is 120 ones.
I need to regroup into 1 hundred and 2 tens.
20 groups of 3 tens is 6 hundreds.
I need to add the regrouped 1 hundred. I now have 7 hundreds.
20 groups of 8 hundred is 16 thousand. There are no regroups to add.
The total of the two partial products is 22, 572.
The product of 836 and 27 is 22, 572.

Pictorial - Jottings

Jottings: multiples of tricky multipliers

7
14
21
28
35
42
49
56
63
70
77
84

Abstract - Written symbolic

```
     1
   2 4
   8 3 6
 x   2 7
 -------
 5 8 5 2
1 6 7 2 0
---------
2 2 5 7 2
---------
 1 1
```

836 x 27 = 22,572

This example shows long division for numbers up to 4 digits - expressing quotients with fractions –

Abstract speaking frame

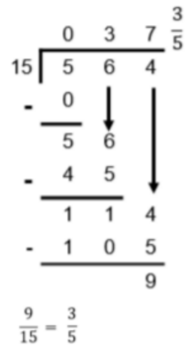
I have a remainder of 9.
This is 9 out of the 15 which I need for another group.
This can be written as a fraction $\frac{9}{15}$.
This can be simplified to $\frac{3}{5}$.
There are $37\frac{3}{5}$ in each of the 15 groups.

Pictorial - Jottings

Jottings: multiples of the divisor

15
30
45
60
75
90
105
120
135
150

Abstract - Written symbolic



$564 \div 15 = 37\frac{3}{5}$

This example shows long division for numbers up to 4 digits - expressing quotients with decimals –

Abstract speaking frame

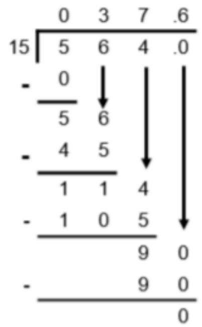
I have a remainder of 9.
I need to regroup the 9 ones into 90 tenths.
How many groups of 15 tenths are in 90 tenths, without regrouping?
I can make 6 groups of 15 tenths.
There is nothing remaining.
There are 37.6 groups of 15 in 564.

Pictorial - Jottings

Jottings: multiples of the divisor

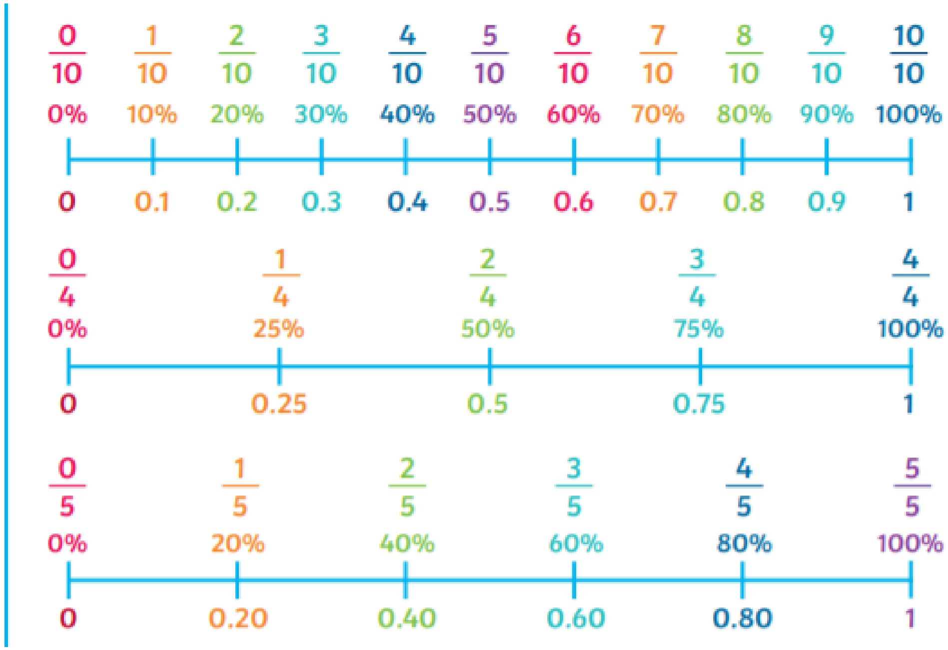
15
30
45
60
75
90
105
120
135
150

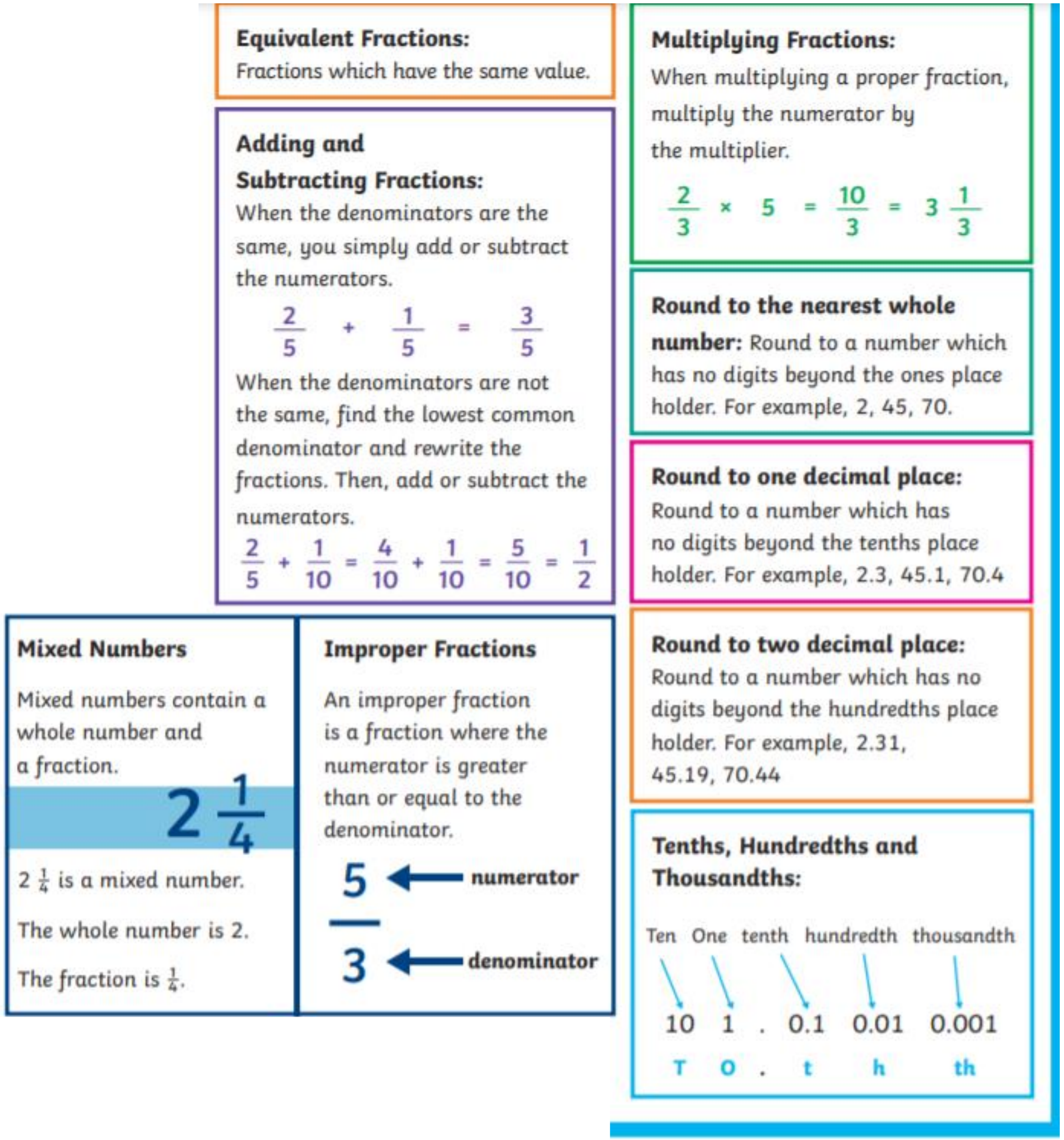
Abstract - Written symbolic



$564 \div 15 = 37.6$

Fractions, decimals and percentages -




Equivalent Fractions:
Fractions which have the same value.

Adding and Subtracting Fractions:
When the denominators are the same, you simply add or subtract the numerators.

$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

When the denominators are not the same, find the lowest common denominator and rewrite the fractions. Then, add or subtract the numerators.

$\frac{2}{5} + \frac{1}{10} = \frac{4}{10} + \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$

Multiplying Fractions:
When multiplying a proper fraction, multiply the numerator by the multiplier.

$\frac{2}{3} \times 5 = \frac{10}{3} = 3\frac{1}{3}$

Round to the nearest whole number: Round to a number which has no digits beyond the ones place holder. For example, 2, 45, 70.

Round to one decimal place:
Round to a number which has no digits beyond the tenths place holder. For example, 2.3, 45.1, 70.4

Round to two decimal place:
Round to a number which has no digits beyond the hundredths place holder. For example, 2.31, 45.19, 70.44

Mixed Numbers
Mixed numbers contain a whole number and a fraction.

$2\frac{1}{4}$

$2\frac{1}{4}$ is a mixed number.

The whole number is 2.

The fraction is $\frac{1}{4}$.

Improper Fractions
An improper fraction is a fraction where the numerator is greater than or equal to the denominator.

5 ← numerator
3 ← denominator

Tenths, Hundredths and Thousandths:
Ten One tenth hundredth thousandth
10 1 . 0.1 0.01 0.001
T O . t h th

See key vocabulary map under 'Curriculum' section on the school website for fractions related vocabulary.

Measurements -

Length

1 kilometre = 1000 metres

1 metre = 100 centimetres

1 centimetre = 10 millimetres

km, m, cm, mm

1 kilometre = 0.62 miles

1 metre = 1.09 yards

1 metre = 3.28 feet

km, m, yd, ft

1 centimetre = 0.39 inches

1 foot = 12 inches

1 yard = 3 feet

cm, in, ft, yd

Capacity

1 litre = 1000 millilitres

1 centilitre = 10 millilitres

l, cl, ml

1 litre = 35.19 fluid ounces

1 litre = 1.75 pints

1 litre = 0.21 gallons

1 gallon = 8 pints

l, fl oz, pt, gal

Mass

1 tonne = 1000 kilograms

1 kilogram = 1000 grams

1 gram = 1000 milligrams

t, kg, g, mg

1 gram = 0.035 ounces

1 kilogram = 2.2 pounds

1 stone = 14 pounds

1 stone = 6.35 kilograms

g, oz, kg, lb, s

Temperature

1° celsius = 33.8° fahrenheit

0° celsius = 32° fahrenheit

°C, °F

Time

1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

hr, min, sec

Currency

1 pound = 100 pence

£, p

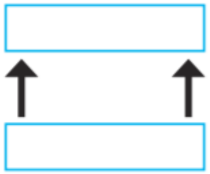
See key vocabulary map under 'Curriculum' section on the school website for measurement related vocabulary.

Reflection and translation –

Translate / Translation

A shape is translated when it is moved without rotating or resizing.

Every point of the shape moves the same distance in the same direction.



Reflect / Reflection

A shape is reflected about a line when it is flipped over a mirror line.

Every point of the shape is the same distance from the mirror line as the same point on the reflected shape.

See key vocabulary map under 'Curriculum' section on the school website for further related vocabulary.

Ratio and proportion –

Ratio

Ratio shows the relative sizes of two or more values.

The ratio of yellow spots to blue spots is 3:2.

Proportion

Proportion is a part or share in relation to the whole.

$\frac{3}{5}$ are yellow spots.

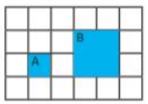
$\frac{2}{5}$ are blue spots.

Scale and Scale Factor

Scaling is used to enlarge or reduce the size of a shape based on the scale factor.

The scale factor represents the ratio of the lengths of the sides of the shape.

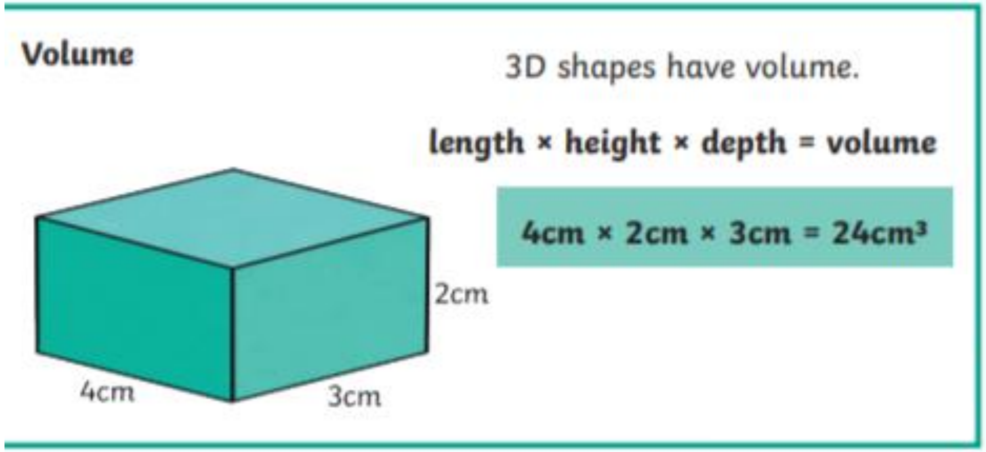
Shape A has been enlarged by scale factor 2 as the length and width of the shape has been doubled.

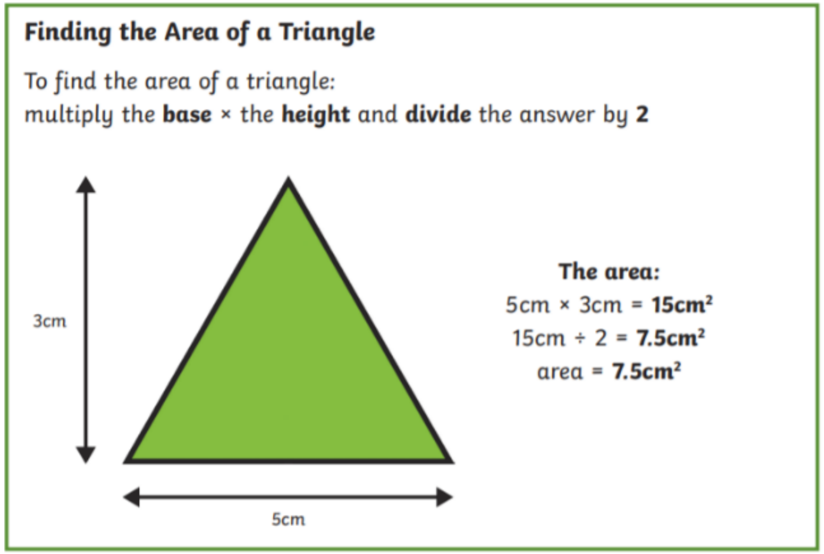


See key vocabulary map under 'Curriculum' section on the school website for further related vocabulary.

Area and Volume –

Area = length x width





See key vocabulary map under 'Curriculum' section on the school website for area and volume related vocabulary.

Factors, multiples and primes -

Factors and Multiples

A multiple is a number that can be divided evenly by a given number.

For example, 12 × 1 = **12**, 12 × 2 = **24**, 12 × 3 = **36**

The multiples of 12 include: 12, 24, 36, 48...

A factor is a number that is multiplied by another number to get a product.

For example, 12 ÷ **1** = **12**, 12 ÷ **2** = **6**, 12 ÷ **3** = **4**

The factors of 12 are: 1, 2, 3, 4, 6 and 12.

Common Factors

A common factor is a number which is a factor of two or more other numbers. For example, 3 is a common factor of 6 and 9.

Common Multiple

A number which is a multiple of a set of numbers. For example, 16 is a common multiple of 2, 4 and 8.

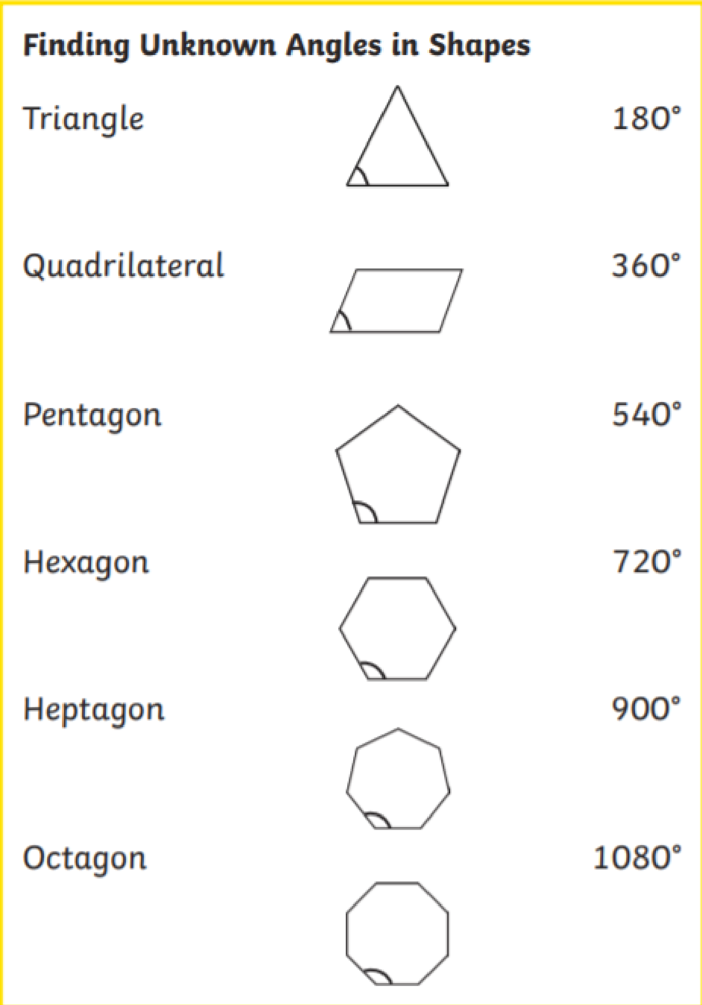
Prime Numbers

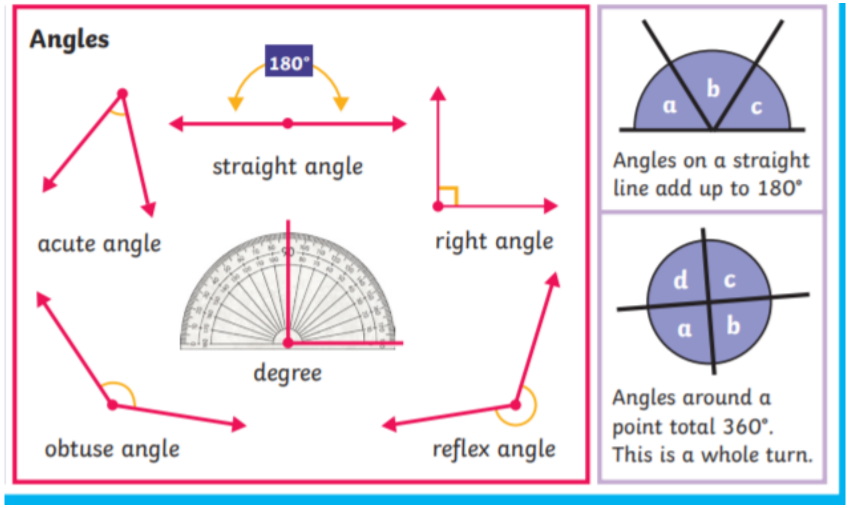
A natural number greater than 1 with no divisors other than 1 and itself.

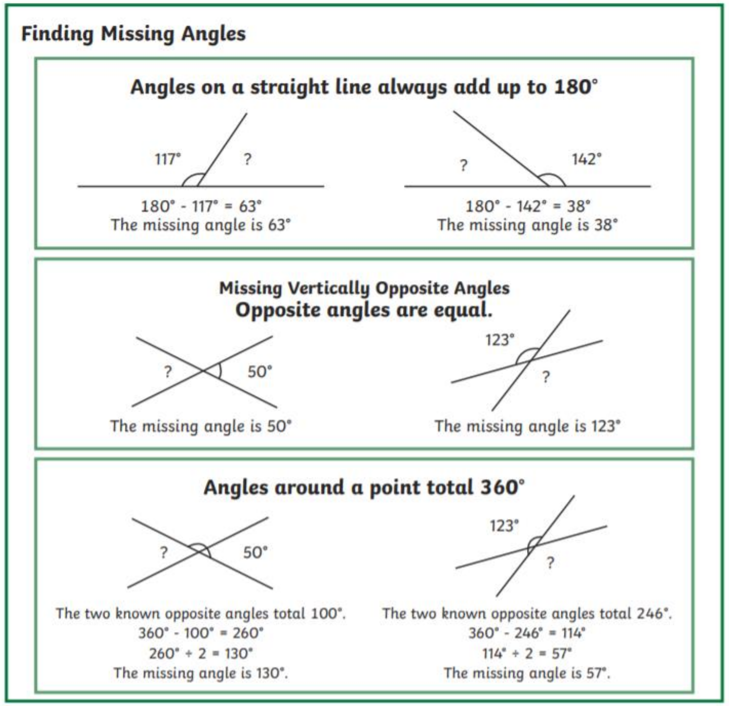
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

See key vocabulary map under 'Curriculum' section on the school website for further related vocabulary.

Angles –

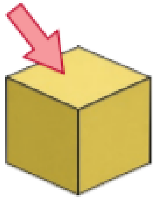

Finding Unknown Angles in Shapes
Triangle
180°
Quadrilateral
360°
Pentagon
540°
Hexagon
720°
Heptagon
900°
Octagon
1080°


Angles
180°
straight angle
acute angle
right angle
degree
obtuse angle
reflex angle
a
b
c
Angles on a straight line add up to 180°
d
c
a
b
Angles around a point total 360°. This is a whole turn.

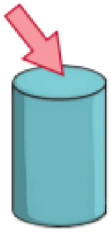

Finding Missing Angles
Angles on a straight line always add up to 180°
117°
?
180° - 117° = 63°
The missing angle is 63°
?
142°
180° - 142° = 38°
The missing angle is 38°
Missing Vertically Opposite Angles
Opposite angles are equal.
?
50°
The missing angle is 50°
123°
?
The missing angle is 123°
Angles around a point total 360°
?
50°
The two known opposite angles total 100°.
360° - 100° = 260°
260° ÷ 2 = 130°
The missing angle is 130°.
123°
?
The two known opposite angles total 246°.
360° - 246° = 114°
114° ÷ 2 = 57°
The missing angle is 57°.

Properties of shapes –

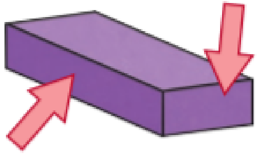
cube
A cube has 6 square faces.



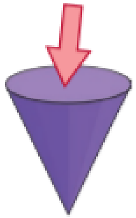
cylinder
A cylinder has two circular faces.



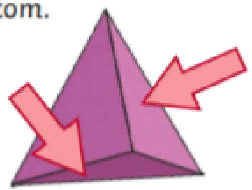
cuboid
A cuboid has 6 rectangular faces.



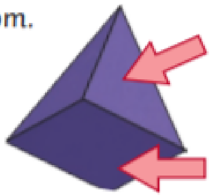
cone
A cone has a circular face.



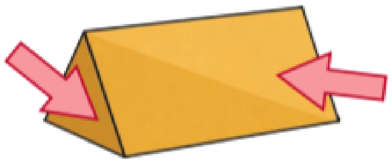
triangular-based pyramid
A triangular-based pyramid has 4 triangular faces. One of the trangular faces is on the bottom.



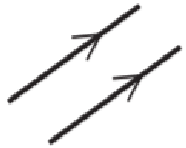
square-based pyramid
A square-based pyramid has 4 triangular faces. It has a square face on the bottom.



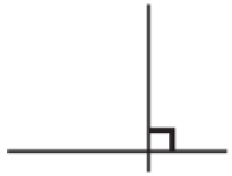
triangular prism
A triangular prism has 2 triangular faces. It has 3 rectangular faces.



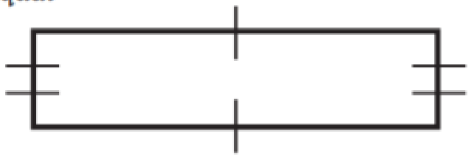
Parallel



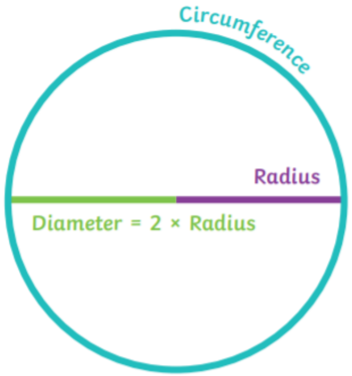
Perpendicular



Equal



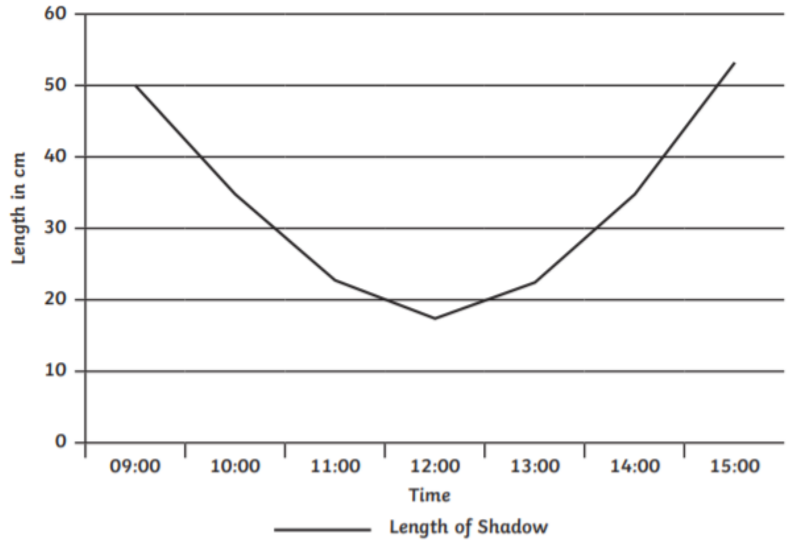
Parts of a Circle



See key vocabulary map under 'Curriculum' section on the school website for further related vocabulary.

Statistics -

Continuous Data



Data that is measured and, therefore, can take on infinite values is continuous.

In continuous data, values between whole numbers can be counted.

In this investigation, it is the length of the shadow that is being measured. This is continuous data because it is possible to record the length as 20.5cm, etc.

Mean

The mean is the average.

5, 5, 6, 4, 7, 3

Add all of the values together.

5 + 5 + 6 + 4 + 7 + 3 = 30

Divide the total by the number of values that you added together.

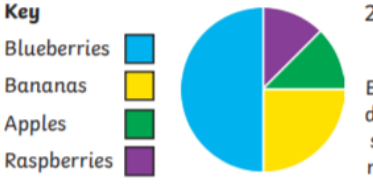
30 ÷ 6 = 5

The mean is **5**.

Pie Chart

Pie charts represent data in a circle divided into segments.

A Pie Chart to Show Children's Favourite Fruit



24 children were asked in total.

Each segment is a different colour or shade, and a key must be included.

See key vocabulary map under 'Curriculum' section on the school website for statistics related vocabulary.